

Bungee Jumping

Bungee Jumping was inspired by the vine jumpers of Pentecost Island in the Pacific Ocean Vanuatu group (formerly the New Hebrides). Each year, the men of the island's tribe construct huge wooden towers, over eighty feet high. They select vines from the jungle which they then tie to their ankles before throwing themselves off the top of the towers.

Inspired by the islanders, modern bungee jumping was invented by members of The Oxford University Dangerous Sports Club. Using nylon braided, rubber shock cord, and dressed in their customary top hat and tails, they performed a four man simultaneous jump from the Clifton Suspension Bridge in Bristol on April 1st 1979.



A. Introduction ●●

Bungee jumps attract those who like danger and excitement. It is an interesting phenomenon for students and can serve as an excellent context for physics lessons.

This module offers activities related to bungee jumping. The following activities are available:

- 1. Video measurement:** measurements on a video of the bungee jump allowing analysis of the bungee jumper motion.
- 2. Data logging:** laboratory experiments to measure force during stretching of the elastic with different masses and force encountered by jumpers on different bungee cords.
- 3. Modelling:** a mathematical model to describe the motion of the bungee jumper.

All activities are offered as Coach 6 Activities in the project Bungee Jumping.

1. Background theory

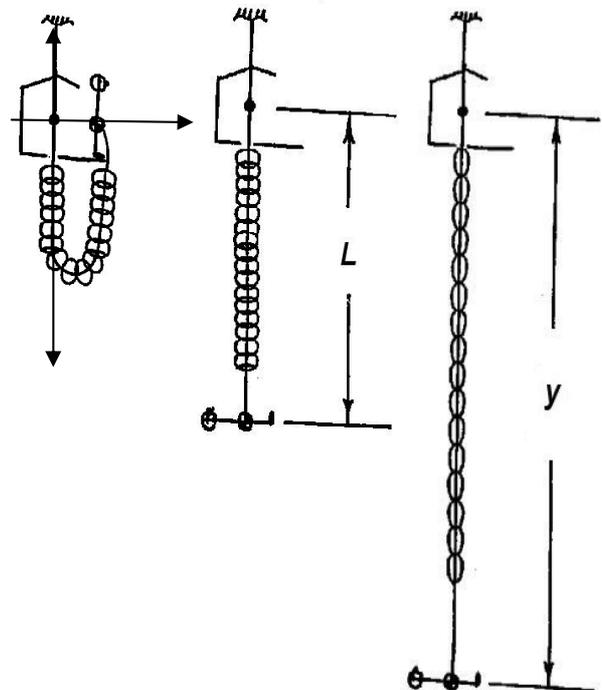
1. INTRODUCTION

A bungee jumper launches himself from a platform to which he is attached from his ankles by a piece of elastic (bungee).

In our model we assume that there is no air resistance and the bungee cord behaves like a linear spring that obeys Hooke's Law.

The bungee jumper of mass m is tied to an elastic cord of length L . The other end of the cord is tied to a fixed point at a platform.

The coordinate system is chosen in the way that the y - axis points straight down and origin is placed at the point where the cord is fixed (as shown on the left figure). The initial position of the jumper is $y = 0$.



The jumper's trip has two distinct phases. First he is in free fall because the cord has not yet extended to its full length L .

When the jumper reaches $y = L$, the cord has extended to its full length

2. FORCES AND MOTION

During the first part of the jump, the jumper is pulled toward earth by gravity. As long as $y < L$ the only force acting on the jumper is

$$F_g = mg$$

At the moment when the bungee cord begins to stretch $y > L$, the upward elastic force from the cord starts to act. Two forces, a constant downwards gravitational force and upward elastic force, act in opposite directions. The total net force is

$$F = mg - k(y - L)$$

where k is a constant describing the stiffness of the cord

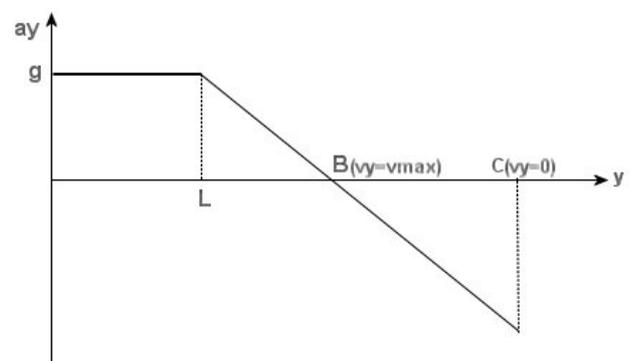
When the jumper passes $y = L$ the gravitational force exceeds the cord's pull and the net force is down. But the elastic force keeps increasing while the weight does not change so the net force is decreasing, until a point where the forces balance and net force is 0. Then the elastic force of the cord pulls the jumper back up.

What are the implications to the motion of the jumper? From Newton's second law, once the cord begins to stretch, the acceleration of the jumper is given by

and from now on it begins to act like a spring pulling him back up. At any $y > L$ the cord is stretched by the amount $y-L$.

$$a = g - \frac{k(y - L)}{m}$$

The figure below show how the acceleration changes during the jump.



When $y < L$ the jumper's acceleration is equal to the gravitational acceleration. Once the point $y = L$ is passed the acceleration decreases linearly with y because of the cord's pull. When $L < y < B$ the acceleration is positive, when $y > B$ it is negative. Until $y = B$ the jumper's velocity increases, he reaches his maximum velocity at $y = B$ and for $y > B$ his velocity decreases. At the point C , which is the lowest point of his jump, his velocity is 0. After reaching this point the jumper moves upwards.

3. ENERGY TRANSFORMATION

The depth of the jump (its lowest point) can be calculated from energy changes considerations. Since we have selected initial position on the platform as the origin, this will be also the reference point for gravitational potential energy.

Standing on the platform the jumper has zero total energy, as his potential energy and kinetic energy are zero. During free fall, some of the jumper's original gravitational potential energy is transferred into kinetic energy. His total energy remains zero

$$y < L \quad E_{total} = -mgy + \frac{1}{2}mv^2 = 0$$

When the bungee cord starts to stretch the potential energy converts also into elastic energy of the cord.

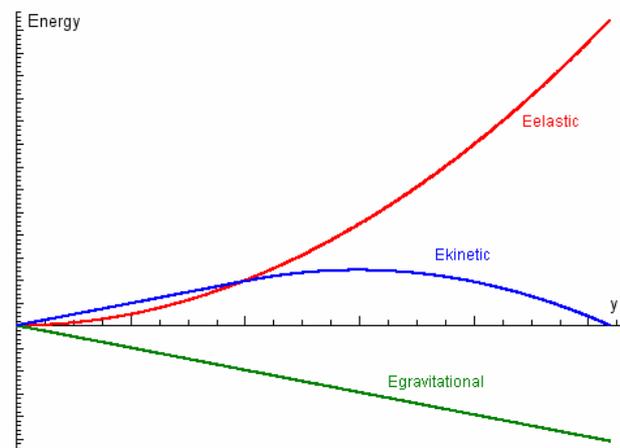
$$y > L \quad E_{total} = -mgy + \frac{1}{2}mv^2 + \frac{1}{2}k(y - L)^2 = 0$$

At the lowest point when the jumper has fallen the maximum distance, the cord is stretched to y_{max} and the velocity of the jumper is zero, and

$$y = y_{max} \quad E_{total} = -mgy_{max} + \frac{1}{2}k(y_{max} - L)^2 = 0$$

The figure shows how the different types of energy change during the jump.

Note that at any point in the fall, the sum of the kinetic and elastic potential energies is equal to the gravitational potential energy lost during the fall. At the lowest point of the jump the loss in gravitational energy is matched by a corresponding increase in the elastic potential energy of the bungee cord.



In our model we have neglected non-conservative forces. If this approximation was correct, the jumper would continue forever to oscillate up and down between the starting and the lowest point. Obviously this is not the case. Air resistance and elastic hysteresis of the cord cause the transfer of energy from the jumper-cord system to the surrounding. And the jumper will finally come to rest.

4. HOOKE'S LAW AND ELASTIC POTENTIAL ENERGY

The work done to stop the jumper is related to the stiffness of the bungee cord. In our simple model the cord acts like a spring that obeys Hooke's Law given by

$$F_e = -k(y - L)$$

where F is the elastic force, k is the spring constant and $(y-L)$ is the stretch of the cord.

The elastic potential energy possessed by the cord

$$E_e = \frac{1}{2} F(y - L) \text{ or by using the stiffness } k \ E_e = \frac{1}{2} k(y - L)^2$$

At the lowest point of the fall the loss of the potential gravitational energy is equal to the elastic potential energy stored in the cord

$$mgy_{\max} = \frac{1}{2} F_e (y_{\max} - L) \text{ or by using the stiffness } k \ mgy_{\max} = \frac{1}{2} k(y_{\max} - L)^2$$

which allows to calculate the value of y_{\max} - the lowest point of the jump.

Hooke's law can also be applied to determine the maximum force experienced by the jumper. If the cord is very stiff (imagine a steel cable) then being stopped by the cord would probably not be significantly better than being stopped by the ground. A cord with more stretch, lower stiffness, gives a "softer ride".

Suppose the person of mass 70 kg wants to jump using a 10-m cord which will stretch another 20 m. The stiffness of the cord can be found:

$$k = 2mg \frac{y_{\max}}{(y_{\max} - L)^2} = 2 * 70kg * 9.81 \frac{m}{s^2} * \frac{30m}{(20m)^2} = 103N / m$$

and the force acting on the jumper is

$$F = k(y_{\max} - L) = 103N / m * 20m = 2060N$$

The 200 % stretch produces a maximum force of 2060 N which is near three times the jumper weight. A 300 % stretch gives $k = 61 \text{ N/m}$ and $F_{\max} = 1831 \text{ N}$ and is only 2.6 of his weight (in other words his acceleration is 2.6 g). This is much softer ride.

The **overload** (weight experienced by a jumper) depends on stiffness of a cord, and of a jumper mass and eventually from a maximum **strain**:

$$\Gamma = \frac{F_{\max}}{mg} = \frac{k(y_{\max} - L)}{mg}$$

But as $mg y_{\max} = \frac{1}{2} k(y_{\max} - L)^2$, then

$$\Gamma = \frac{k(y_{\max} - L)}{mg} = \frac{2y_{\max}}{y_{\max} - L} = 2 \frac{L + \Delta L}{\Delta L} = 2 \left(\frac{L}{\Delta L} + 1 \right) \quad \left(\text{when } \frac{\Delta L}{L} = \text{strain} \right)$$

It can be seen that overload cannot be lower than 2, even for very large elongation.

In reality the bungee cords do not obey the Hooke's law and demonstrate variable stiffness over their range of use. Paul G. Menz in his article "The Physics of bungee jumping" [2] suggests approximating the force – elongation graph with two segments with different slopes as shown on the next figure.

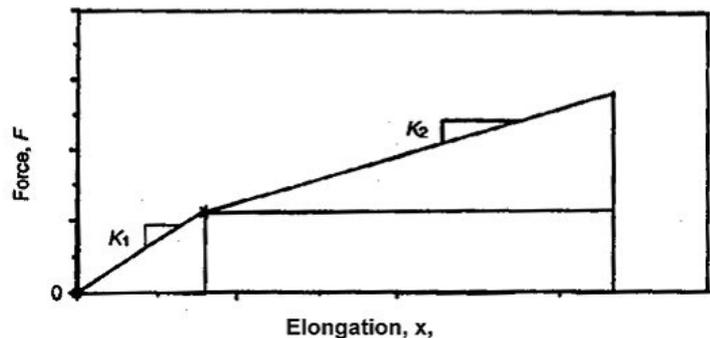


Figure. The Force against elongation approximation graph [2].

5. RISKS IN BUNGEE JUMPING

The influence of bungee jumping on the jumper body depends on the stage of his motion:

1. During **Free fall** stress hormones, like beta-endorphin, growth hormone, prolactin, testosterone, and adrenalin, are released. It explains the feelings of exhilaration and well being that often occurs and last for some days after a single jump.
2. **Body deceleration** - the intensity of deceleration depends on the kind of cord used and the altitude of the jump. The more intense deceleration the more is the risk of injuries. At this stage the hydrostatic pressure in the blood vessels of eye becomes dangerously high. Because the head-down position a sudden rise in intrathoracic pressure also occurs and it is one of the major damage event.
3. **Upward movement** in a head-down position is responsible for a further increasing in intrathoracic and head blood vessels pressures. According to some aviation studies, forces exceeding -3g are able to cause hemorrhages. [5]

Because of the number of factors involved, it is difficult to predict how much acceleration a certain individual can withstand. Tolerance is related to the rate of onset of acceleration and to the duration of exposure. Individual tolerance depends on factors such as the height of the person, age, elasticity of the blood vessels, training, the responses of the heart and blood vessels, and on health.

A typical person can handle about 5g (50 m/s²) before loss of consciousness (the blackout) occurs. Through the combination of special g-suits and efforts to strain muscles - both of which act to force blood back into the brain - modern pilots can typically handle 9g (90 m/s²). Resistance to "negative" or upward gees, which drive blood to the head, is much less; typically in the 2-3g (20 to 30 m/s²) range the vision goes red, probably due to capillaries in the eyes bursting under the increased blood pressure.

2. Pre-requisite knowledge required

- Equations of motion
- Newton's laws
- Force and acceleration (Free fall)
- Gravity Force, Elastic Force
- Hooke's Law
- Kinetic, Potential and Elastic Potential Energy

3. Science concepts developed in the module

These depend on the educational approach (see also Section B.1) concepts such free fall, energy transfer, energy conservation, Hooke's law and non-linear behaviour of elastic cords can be introduced or applied in this module.

4. Other useful information

T. Martin, J. Martin, "The physics of bungee jumping", *Phys. Educ.* **29**, 247-248 (1994)

P. Menz, "The Physics of bungee jumping", *Phys. Teach.* **31**, 483-487 (1993)

K. Ernst, Szaleństwo na linie, *Wiedza i Życie*, Sierpień 1999

Salters Horners Advanced Physics, Heinemann Educational Publishers, 2000,

Mario Cesar Moreira de Araujo, Marcelo Riccio Facio, *Sports Medicine*, at <http://www.medstudents.com.br/sport/sport7.htm>

Internet resources:

<http://www.netipedia.com>

<http://www.challenge.nm.org/sti/projects/bungee.html>

B. Didactical approach ●●

1. Pedagogical context

The proposed Bungee Jumping module can be used in two different educational approaches:

1. as a context module or
2. as an application module

In the first approach concepts can be defined and clarified, new terms can be introduced. Students are guided to invent concepts and principles that help them answer questions. Teachers employ enquiry approaches in the activities.¹

In the second approach students use the concepts that were already introduced and apply them to new situations. The goal is to have students generalize the application of their knowledge. This application of the principles and concepts leads to further understanding of the theories and the models.

In both approaches this module places students' learning in an environment in which physics is met in real life.

¹ Similar approach of a context-led physics course has been introduced in United Kingdom in courses like Advancing Physics, or Salters Horners Advanced Physics.

2. Common student difficulties

- Common student difficulties concerning Kinematics and Dynamics:
- Differentiating the concepts of position, velocity and acceleration.
- Making, using and interpreting graphs with time as the variable plotted on the x-axis.
- Students make connection between the kinematical concepts, their graphical representation and the motions of real objects. They plot position and velocity as the path of motion.
- Distinguishing between slope and height of a graph.
- Separating slope from path of motion.
- Interpreting changes in height and changes in slope (e.g. is the object slowing down, which motion is slowest).
- Understanding the physical significance of the sign of a body's velocity.
- Understanding that it is possible to have zero velocity and non-zero acceleration, or non-zero velocity and zero acceleration.
- Understanding that the direction of acceleration relative to velocity determines whether an object speeds up or slows down.

- Understanding that a body can have a positive velocity and negative acceleration (or the reverse) simultaneously.
- Identifying the forces acting on an *isolated* object in a many body system.
- Determining the direction of tension in a rope in a many object system.
- Belief that the motion implies a force (so no movement means no forces).
- Many students assume a linear relationship between force and velocity.
- Forces are properties of objects, rather than manifestations of interactions between objects; for example, weight is a property of an object, rather than reflecting the force of gravity between the Earth and the object.
- Gravity depends on height
- Heavier objects fall faster than light objects.
- Only living things can exert forces on other things.
- Connecting idealized model with non-ideal reality (esp. regarding friction!).
- Friction is not a force; it only slows things down.
- Concept of conservation.

REFERENCES:

1. Beichner, R. J., *Testing student interpretation of kinematics graphs*. American Journal of Physics, 62, 750, (1994)
2. F. M. Goldberg and J. H. Anderson, Student difficulties with graphical representations of negative values of velocity, Phys. Teach. 27, 254-260 (1989).
3. R. F. Gunstone, Student understanding in mechanics: A large population survey, Am. J. Phys. 55, 691-696 (1987)
4. A. Halloun and D. Hestenes, *Common-sense concepts about motion*, Am. J. Phys. 53, 1056-1065 (1985).
5. McDermott, Lillian C., Mark L. Rosenquist, and Emily H. Van Zee. , American Journal of Physics, 55, 503, (1987)
6. Student Difficulties in Physics Information Center, Montana State University, Bozeman
<http://www.physics.montana.edu/phised/misconceptions/>
7. Galileo Project materials, more information about this project can be found at <http://galileo.harvard.edu>

3. Evaluation of ICT

The specific qualities of ICT which benefit student learning are specified below per a type of activity.

VIDEO

Videos clips and hypertext offers the possibility to present selected information. Discussion around presented materials can help teachers to pre-assess student knowledge.

Activity 0. A little bit of history (optional)

This activity presents short video clips which illustrate different bungee jumps as performed in reality: original African jumps by the inhabitants of Vanuatu and jumps from a high crane and a helicopter (with a very long cord). Additional a short text about history of bungee jumping is included.

This activity can be used as an introduction to the topic, to trigger student's interest and stimulate curiosity. In order to make bungee jumping exciting, but safe, detailed study of physics of the motion is necessary.

VIDEO MEASUREMENT

In the video-measurement activities students are able to consider events, which happen outside the classroom. The data are collected in the form of points by clicking in successive (selected) frames on (a point of) a moving object. The video data can be plotted in a graph or viewed in a table. When scanning the data in the diagram the corresponding video frames are shown. This helps students to bridge the gap between the concrete visual display of a motion event and its abstract graphical representation.

Activity 1. Bungee Jump

The video of the bungee jump from a high crane allows exploring in details the jumper's motion and analysing the different phases of his motion.

Position and time data are collected by clicking on the location of the jumper in each frame of the video. The position data are plot immediately and can be used for calculating the velocity and the acceleration experienced by the jumper. By using the *Scan* option students can compare the real motion of the jumper with its motion graphs.



The jumper motion can be divided into three stages:

1. stage 1 – free fall, until a cord is stretching
2. stage 2 – decelerated fall caused by growing cord tension
3. stage 3 – pulling up a jumper by stretched cord.

During the first stage (neglecting air resistance) jumper is moving with gravitational acceleration. As each parts of his body are moving identically, interactions among them disappeared, and the jumper experiences **weightlessness**.

While the cord tension is growing, jumper feels growing weight of his body, which attains the normal value when the net force (and therefore deceleration) is zero. At this moment the jumper's velocity reaches its maximal value.

Then the cord stretches further until the jumper stops. Then the cord is exerting maximal force several times bigger than his weight. It causes the **overload** expressed as a multiple of gravitational acceleration g . The value of overload is different from the actual acceleration created by the net force whereas overload is created by the cord tension.

Maximal acceleration read from the obtained graph is 15 m/s^2 (upward) – or $1.5 \times g$. It comes from directed upward acceleration created by cord tension, reduced by directed downward acceleration of free fall (about 10 m/s^2). An acceleration due only to tension force is about 25 m/s^2 , and then overload (weight experienced by a jumper) is about **$2.5 g$** .

From the graphs also other parameters necessary for analysis can be read:

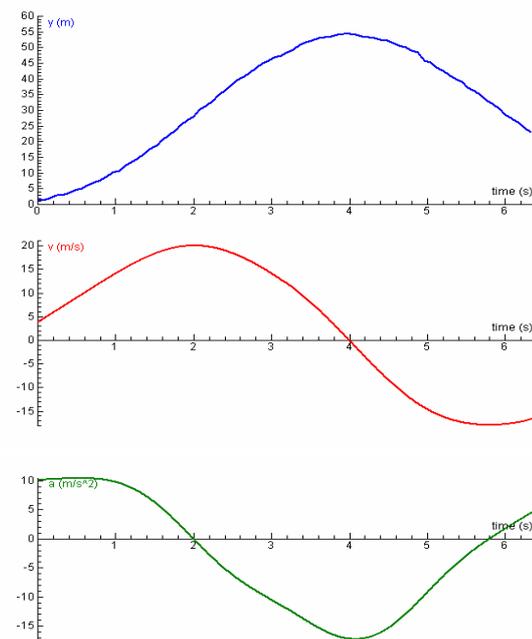
Cord initial length $L = 11\text{m}$

Maximal cord elongation - $y_{\text{max}} - L = 42\text{m}$.

Stiffness of a cord and jumper mass are related. Therefore one of these values must be known.

Let assume that the jumper mass is 70kg . Then the stiffness of the cord is 40 N/m .

The depth of the jump – its lowest point can be calculated from energy changes considerations (see also 1. Background theory 4. *Hooke's Law and Elastic Potential Energy*).



On the video jumper weighting 70 kg jumps using an 11-m cord, will stretch 42 m (nearly 4 times). The stiffness of the cord can be found.

$$k = 2mg \frac{y_{\max}}{(y_{\max} - L)^2} = 2 * 70\text{kg} * 9.81 \frac{\text{m}}{\text{s}^2} * \frac{53\text{m}}{(42\text{m})^2} \approx 40\text{N} / \text{m}$$

The maximum force acting on the jumper is

$$F_{\max} = k(y_{\max} - L) = 40\text{N} / \text{m} * 42\text{m} = 1680\text{N}$$

which is near two and a half times the jumper weight. (It confirms former estimation from acceleration graph.)

DATA LOGGING

The benefits of data-logging activities are coming from possibilities of direct measurement of force, display of measured data in graphs and tables, calculations of the energy values and plotting the energy changes. Additional value of this experiment can be learning how to calibrate the force sensor.

The goal of data-logging activities is to design the bungee jumps that will allow a jumper to have safe but exciting jumps. The jumper should not decelerate too rapidly and should not touch the ground but come as close to the ground as possible.

Activity 2. Safe Bungee Height

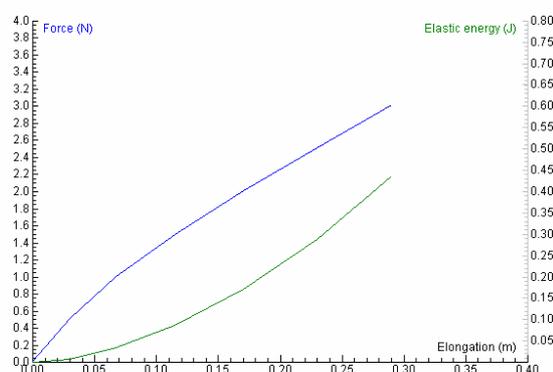
The aim of this activity is laboratory measurement of elastic cord stiffness. In this experiment force is measured when no mass hangs at the elastic cord and each time a mass is added. Students investigate how the position of the end of elastic cord changes with an increasing load on it.

In the second part of the activity students have to create a graph of elastic energy E stored at various values of extension. Based on this graph and the graph of the potential energy lost by the jumper they have to predict the height from which the bungee jumper must be lunched in order to achieve a jump that is both safe and exciting.

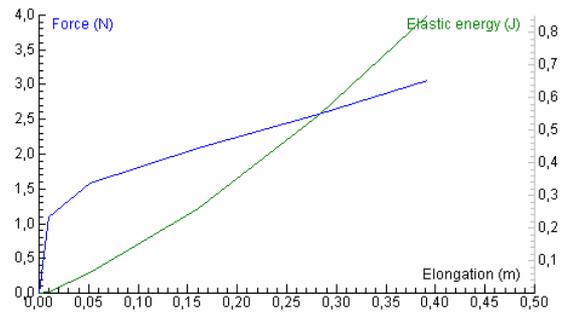
For an elastic cord that obeys Hooke's Law the force against extension graph is a straight line. The slope of this graph allows determining the elastic stiffness of the cord.

The elastic potential energy can be found from

$$E_{\text{elastic}} = \frac{1}{2} F(y - L).$$



For an elastic cord (e.g. rubber cord) which does not obey Hooke's Law, the graph is not linear. The elastic energy stored in such cord can be found by finding the area under the relevant section of the graph (integration).



Calculation of extension and strain is important as it allow estimating the height from which the bungee jumper can be safely launched to stop above ground and estimating overload.

It is calculated using energy conservation, and the fact that when the bungee jumper falls then at the lowest position his velocity becomes zero, the loss of his gravitational potential energy is equal to the energy stored in the stretched elastic. On a graph of the loss of elastic energy stored in the elastic cord and gravitational potential energy of the jumper against position of the jumper this moment is given by the intersection point of the energy graphs.

Activity 3. Safe Bungee Cord

This experiment focuses on measuring and graphing force in time acting on different bungee cords during the jump. The maximum force acting on the bungee can be read directly from the graph. Here also overload experienced by a "jumper" can be calculated.

MODELLING

The Modelling activities enable to use and create numerical models of continuous changing systems. In such models the dynamic change of the system is described step-by-step.

Modelling allows creating models of complex phenomena and solving realistic problems which are quite difficult to solve analytically at the school level. The formulation of such numerical models is often rather simple and conceptually easy to understand, and may bear little relation to the mathematical difficulty of getting a solution.

The model results can be then compared to experimental data (from measurement or video measurement). The model can be easily modified or a parameter can be changed so students can test their hypothesis and make links between the real experiment and the theoretical model.

Activity 4. Bungee Challenge

The purpose of this activity is to create a dynamical model of the bungee jumper motion and solve the problem of determining which elastic cord is best suited for the given weight.

In the model the position y and velocity v of a moving object can be found by numerical integration:

$$\frac{dy}{dt} = v \quad \text{and} \quad \frac{dv}{dt} = a$$

and the acceleration a can be found from the Newton's second law $a = \frac{F_{net}}{m}$

where F_{net} is the net force acting on the object of mass m . By defining the force acting on the object its motion is fully described.

In this activity students start with a model of the constant accelerated motion² with the acceleration $a = 10 \text{ m/s}^2$. They have to revise this model and build up a new model by adding the formulas for calculating acceleration and forces acting on the jumper.

The total net force acting on the jumper is

$$F_{net} = F_g \quad \text{when } y \leq L \quad \text{and} \quad F_{net} = F_g + F_e \quad \text{when } y > L \quad \text{where}$$

the gravitational force is $F_g = mg$ and the elastic force is $F_e = -k(y - L)$.

By studying the model and experimenting with the bungee cords stiffness k (using the Simulation option) students find which cord is the most appropriate for a safe bungee jump.

When students compare their models with the real data they can notice that the model does not describe well the reality, the bungee movement does not stop. The model can be refined for example by adding the air resistance force. In most cases it may be assumed that the air resistance force is proportional to the square of velocity and depends on the shape and dimensions of a moving body (its projected area).

$$F_{air} = -\text{sign}(v) * s * v^2 \quad \text{where } s \text{ is a projected area}$$

The direction of the air resistance is opposite to the direction of motion (direction of the velocity).

² It is assumed that such a model is already known by students.

4. TEACHING APPROACHES

The four activities presented here offer distinctive but complementary insights into the science involved in this topic. For the activities to be effective for teaching and learning, it is helpful for teachers to consider two types of skills in using the software tools:

- **Operational skills** which concern the manipulation of the computer hardware and knowledge of the features in the software.
- **Procedural skills** which concern the manner in which the software tools are employed in the lesson context for the purpose of achieving learning benefits. A dominant aspect of these skills is the development of an inquiring approach to the analysis and interpretation of data and to making links with previous knowledge.

Such skills are important for the preparation of pupils for the activities, and the activity sheets below each contain indications of the skills needed for the particular activity.

For the teacher, there are further *pedagogical skills* which contribute to the effectiveness of the activities:

1. Clarity of learning objectives for each activity.
2. Understanding of the special value of the ICT method and exploiting its full potential in purposeful ways.
3. To manage the activity in a way which promotes 'appropriate' rather than 'indiscriminate' use of ICT.

4. To integrate the learning from each activity to develop pupils' understanding of the topic.

The development of the last of these is a particular aim of the IT for US Project, and the activities presented have been specially selected to illustrate how integration might be achieved. Comparisons of the observations and results of each activity form a central role in this integration process. For example:

Use the results from the video measurement (activity) and data logging experiment (activity 2) to contribute to a discussion about energy transformations.

Use the two data logging experiments (activity 2 and 3) and model (activity 4) to contribute to a discussion about safety and risks of bungee jumping.

Use the video measurement (activity 1) and a model (activity 4) to discuss the stages of jumper motion and forces acting on the jumper during its motion.

In these, the graph is a key tool in facilitating comparisons and interpretations and skills with graphs generally provide a common thread in exploiting IT for US activities.

The management of the classroom setting also has an important influence on the successful integration of activities. When access to computer equipment is scarce it is likely that the teacher will wish to present the activity as a demonstration in a didactic manner. In this mode, the teacher can give strong guidance to pupils' thinking about the comparisons between the

activities.

Alternatively, pupils could perform the activities in small groups of three or four pupils, each group engaged on a different activity.

Integration might be achieved by each group making a presentation of their results to the whole class. In chairing these presentations the teacher can prompt discussion of the significant findings of each group.

It is worth considering that all the activities may be used in a variety of learning contexts.

Although the activities have been designed to provide complementary experiences, it is not essential to use all of them; two or three activities might be chosen according to how well they suit the needs of teachers and pupils in a particular context. In

varying conditions between schools and within schools at different times of the year or different stages in the curriculum, needs and appropriateness are likely to change; for example, data-logging equipment might not be available at the time of need, an individual pupil might need a revision or extension activity, an enrichment activity might be required to occupy some spare time, a quick activity might be needed if time is scarce. The overlapping features, such as graphical presentation, between the activities allows them to be used to a certain extent as alternatives, but their distinctive features also allow them to be used as complements to each other. The table below summarises the distinctive potential learning benefits of each. It is a useful guide to the special value of each ICT activity

Activity	Potential learning benefits, 'ICT value'
Video measurement	Allows detailed analysis of the motion of the real bungee jumper. Graph of position versus time is created during the measurement. This graph can be used for calculating the velocity and the acceleration experienced by the jumper. Graph analysis tools facilitate detailed investigation of data. The motion of the jumper can be easily compared with its graphical representations.
Data logging	Graph of force changes is displayed during the experiment. Graph analysis tools facilitate detailed investigation of data. Processing tools allow calculation of energy changes.
Modelling	The model demonstrates how the relevant physical principles can be expressed in simple stages using mathematical formula. The model calculates position, velocity, acceleration and force acting on a bungee jumper. The effect of altering parameters such the bungee cord stiffness k is investigated. The model data can be compared with experimental data.

C. Student Activities ●●

ACTIVITY 0. A LITTLE BIT OF HISTORY...

Learning Objectives:

1. To introduce students to the problem of the bungee jumping
2. To interest students with the subject and stimulate their curiosity

APPLIED ICT TECHNOLOGY:
VIDEO MEASUREMENT

STUDENT LEVEL:
AGE 17

RECOMMENDED SETTINGS:
STUDENT ACTIVITY

Operational Skills:

- Opening Coach program and Coach Activities
- Starting/Stopping the video clips

Materials:

Video clips of bungee jumps.

Activity method:

Students should watch the video clips and read the included text.

Questions:

Discussions might include questions like:

- What are the forces acting on a bungee jumper?
- What acceleration experiences a bungee jumper?
- What determines how far the jumper will fall before bottoming out? See what happened to some jumpers in the African jump video clip)

To be able to answer these questions students need to perform activities included in this module.

Coach 6 Activity:

0. A little bit of history

ACTIVITY 1. BUNGEE JUMP

Learning Objectives:

1. To interpret $y(t)$ graph with respect to the visualized motion of the bungee jumper
2. To determine and interpret $v(t)$ and $a(t)$ graphs
3. To determine the maximal acceleration occurring during a jump
4. To interest them with the subject and stimulate their curiosity

APPLIED ICT TECHNOLOGY:
VIDEO MEASUREMENT

STUDENT LEVEL:
AGE 17

RECOMMENDED SETTINGS:
STUDENT ACTIVITY

Operational Skills:

- Making measurements on the Video Screen
- Using the Scan, Derivative, and Slope options
- Creating new diagrams

Procedural Skills:

- Analysing data using a graph
- Reading values/slopes
- Evaluating measurement quality
- Using time bonus

Materials:

Video clip of the bungee jump. This video originates from the Video Clips Collection of Physics Education Group Munich University

Activity method:

1. In the Data Video Window you see the first frame of the video Bungee Jump.
2. Play the video clip.
3. Carry out measurements on the position of the jumper. The movie is already scaled and the graph of Vertical position versus time is prepared. Notice how the coordinates system is positioned in the Video screen.
4. Analyse the resulting graph: What is the initial position of the jumper?
5. Create a velocity versus time $v(t)$ graph (use the *Derivative* option) and

place the graph in the middle, empty window.

6. Create an acceleration versus time $a(t)$ graph and place it in the bottom, empty window.
7. Plot acceleration against position graph and describe how the acceleration changes in function of position.

Questions:

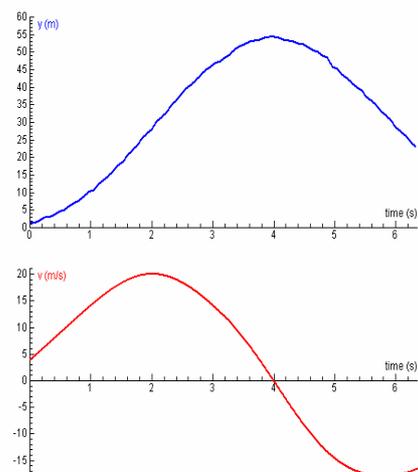
- What is the initial position of the jumper?
- What is the maximal (lowest) position of the jumper when its bungee cord is the most stretched?
- How long does the jumper fall to reach this lowest point?
- What determines how far the jumper will fall before bottoming out?
- What is the initial velocity of the jumper?
- What is the maximum velocity of the jumper?
- What happens to the jumper when the velocity is zero?
- Which section of the motion would you call free fall?
- What is the acceleration at the lowest point? What is the acceleration direction?
- What are the forces acting on a bungee jumper? (Draw a free-body diagram).
- What is net force acting on the jumper when his deceleration reaches maximum?

Analysing activities:

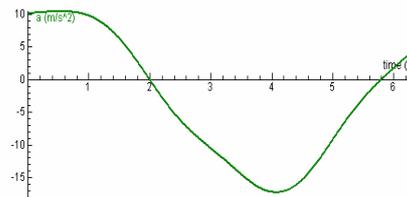
The video clip shows a bungee jump from a high crane. Students collect position and time data from the video screen. The $y(t)$ graph is already prepared in the activity.

They create $v(t)$, $a(t)$ graphs (e.g. by using the Derivative option) and analyse these by looking at initial values, zero points, maximal values and slopes.

The exemplary data are shown in the figures at the right.



The Scan option can be used to analyse all the motion graphs and the video motion at the same time and observe relationships between the position, velocity and acceleration and phases of the jumper motion.



Additionally the graph of $a(y)$ can be created. This graph allows analysing how the acceleration changes during the jump. Based on the graphs students try to describe forces which are acting on the jumper. Also the energy transformations should be discussed.

Hints and tips:

To get nice smooth graphs of $v(t)$ and $a(t)$ the Smooth option, and Smoothed derivatives could be used.

Coach 6 Activity:

1. Bungee jump

ACTIVITY 2. SAFE BUNGEE HEIGHT

Learning Objectives:

1. To interpret force against extension graph created during stretching the elastic cord with different masses
2. To evaluate if the elastic cord obeys the Hooke's Law
3. To know and determine the energy stored in the stretched elastic cord
4. To understand the potential and kinetic energies of a bungee jumping
5. To apply energy conservation for determination of the maximum height for a safe but and exciting (close to the ground) bungee jumping.

APPLIED ICT TECHNOLOGY:
DATA LOGGING

STUDENT LEVEL:
AGE 17

RECOMMENDED SETTINGS:
STUDENT ACTIVITY IF
ENOUGH EQUIPMENT
AVAILABLE
OTHERWISE TEACHER LED
STUDENT ACTIVITY

Operational Skills:

- Connecting sensors and interfaces
- Choosing logging parameters
- Collecting data in the manual measurement mode
- Calibration of force sensor

Procedural Skills:

- Active observation during real-time logging
- Evaluating measurement quality
- Analysing data using graph
- Reading values/slopes
- Creating new diagrams
- Reading values/slopes
- Using time bonus

Materials:

- Interface (CoachLab II/II⁺)
- Force sensor
- Stand and 2 clamps
- Elastic cords of 50 cm length
- Metre ruler
- Hanger and Masses (50 or 100 g depends on the elastic cord)
- Bungee jumper (a toy, or plastic bottle filled with water)

Elastic of 50 cm long is attached to the force sensor which is clamped to the stand. The metre ruler is clamped in a vertical position behind the spring. The light pointer can be used to help students to take position readings accurately. The masses hang at the other end of the elastic cord.



Activity method:

1. Set up your experiment as shown on the photo.
2. Clamp the force sensor to the stand in such a way that it will be stable.
3. Attach one end of the elastic to the force sensor.
4. Clamp the ruler in a vertical position behind the elastic.
5. You can attach the light pointer to the bottom of the elastic. Such indicator helps you to take accurate position readings.
6. Connect the force sensor to input 1 of the CoachLab II interface.
7. You are going to measure the force and type in the position of the bottom of the elastic.
8. Your first measurement should be done when no mass hangs on the elastic.
9. Start your measurement by clicking the green Start button. The measured force value is thus recorded.
10. Type in position value (for the cord of 0.5 m long this value should be 0.5 m). The first point appears in the diagram.
11. Attach one piece of the mass to the end of the elastic cord.

12. Click the green button and record the resulting force. Type in the resulting length.

13. Attach the next mass and repeat your measurement.

14. Continue until the elastic cord is approximately 1.5 long.

Questions:

For the first part of the activity students concentrate on the force versus extension graph, for this part the following questions should be used:

- What kind of relationship does exist between the force and the elastic extension?
- Read from your graph how big the extension of the elastic is if the force is ... (give reasonable value here)?
- Which extension would you expect if the force on the spring will be 100N? Explain.

The linear (Hooke's Law) and non-linear behaviour of the elastic should be discussed based on the given Force versus extension graph.

The second part of the activity utilizes energy conservation and the fact that when the bungee jumper falls then at the lowest position his velocity becomes zero, the loss of his gravitational potential energy is equal to the energy stored in the stretched elastic. Questions concerning the energy conservation and energy transfer should be asked here.

Analysing activities:

Students measure the force acting on the elastic cord first when no mass hangs at the elastic cord and then each time a mass is added. They use manual measurement, the force value is measured by the force sensor and the position of the end of the cord y is read from a metre and typed in. The elastic extension = $y - L$ (where L is the length of the unstretched elastic cord) is calculated in the program. The graph shows force against the extension of the elastic cord.

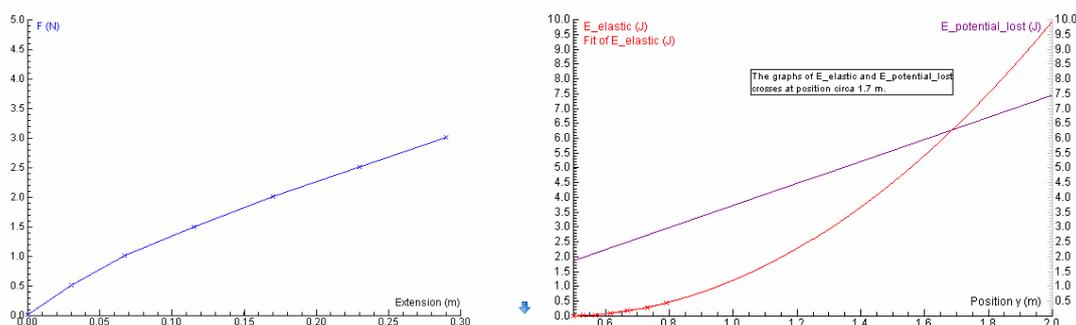
For an elastic cord that obeys Hooke's Law the force against extension graph is a straight line. The slope of this graph allows determining the elastic stiffness of the cord. The elastic potential energy can be found from.

$$E_{elastic} = \frac{1}{2} F(y - L)$$

For an elastic cord (e.g. rubber cord) which does not obey Hooke's Law, the graph is not linear. The elastic energy stored in such cord can be found by finding the area under the relevant section of the graph (integration).

To be able to find the height from which the bungee jumper must be launched, students create a new graph of the loss of the elastic energy stored in the elastic and gravitational potential energy of the jumper against position of the jumper (both should be plot in the same diagram). The potential energy of the jumper falling a distance y is given by $E_g = mgy$.

The moment when the gravitational potential energy lost is equal to the energy stored in the elastic cord is indicated by the intersection point of the energy graphs. A few centimetres should be added to make sure that the jumper does not hit the floor. Students should present their calculations supporting the proposed drop height before any actual testing takes places.



Exemplary data:

left force against extension, right elastic and the potential energy lost against position.

The graph of the elastic energy consists of only a few points. The Function fit option with the quadratic fit should be used to interpolate and extrapolate the graph. Unfortunately extrapolation in Coach is also used for the $y < L$ which does not make sense for the loss of the elastic energy. When $y < L$ the elastic energy is 0. This can be solved by setting the graphs starting at $y = L$.

Hints and tips:

Stand should be well clamped to make a stable support.

The Force sensor should be calibrated in such a way that it displays 0 when the elastic hangs on it, and shows positive values when the mass hangs on it.

The bungee cord should be very flexible. The elastic and masses should match so that the extension has a reasonable size.

Measurements in this activity can be performed without a force sensor by typing manually as well as the mass of the hanger and its position.

Coach 6 Activity:

2. Safe bungee height

ACTIVITY 3. SAFE BUNGEE CORD

Learning Objectives:

1. To be able to interpret the force-time graphs
2. To be able to determine maximum deceleration occurring during the jump

Operational Skills:

- Connecting sensors and interfaces
- Choosing logging parameters
- Collecting data in the time-based measurement mode
- Calibration and zeroing of force sensor

Procedural Skills:

- Active observation during real-time logging
- Evaluating measurement quality
- Analysing data using graph
- Reading values/areas under the graph
- Creating new diagrams

Materials:

- Interface (CoachLab II/II⁺)
- Force sensor
- Stand and clamp
- Elastic cords of different stiffness and the same length
- Bungee jumper of circa 400 g mass (a wooden block, a small toy doll or a plastic bottle filled with water)

Warning:

Do not use too heavy jumper or too stiff cord to avoid destroying the force sensor.

APPLIED ICT TECHNOLOGY:
DATA LOGGING

STUDENT LEVEL:
AGE 17

RECOMMENDED SETTINGS:
STUDENT ACTIVITY IF
ENOUGH EQUIPMENT
AVAILABLE
OTHERWISE TEACHER LED

Activity method:

1. Clamp the force sensor to the stand in such a way that it will be stable.
2. Attach one end of an elastic cord to the hook of the force sensor.
3. Tie the other end of the cord to your bungee jumper.
4. The cord length should be so that the jumper does not hit the floor when dropped.
5. Connect the force sensor to input 1 of the CoachLab II interface.
6. Lift the jumper to the height of the force sensor hook. The elastic cord should be hanging to the side.
7. Start your measurement by clicking the green Start button.
8. Hold the jumper motionless for one second, and then release it so that it falls straight down with a minimum of rotation. The measured force appears in the force versus time graph.
9. Repeat the experiment for each cord. Remember to release the jumper from the same initial position.

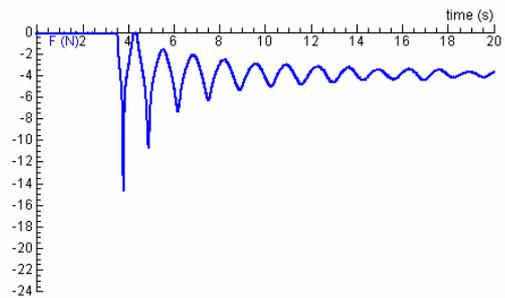
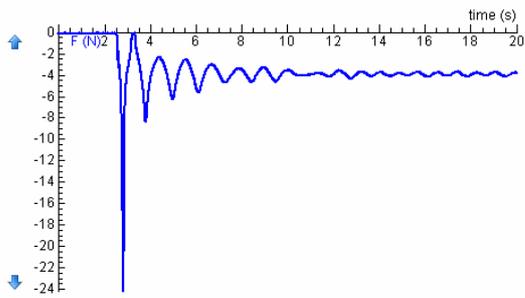
Questions:

- What is the maximum force acting on the bungee jumper for each cord?
- Which cord does produce the greatest force and which the least force?
- What is the maximum deceleration for each cord?
- What is the time interval needed to stop the motion of the jumper for each cord?
- Which of the cord is the safest for the bungee jumper? What can you tell about the stiffness of this cord?

Analysing activities:

In the experiment students obtain different shaped curves of force versus time for different cords. Students should describe (and save) data for each "jump" and clearly indicate which cord or materials was used, what was the amount of force acting on the jumper.

The maximum deceleration encountered by the jumper is reached when the net force acting on the jumper is a maximum. If this acceleration is too great, it may cause damage to the jumper. Based on the maximum force students can calculate the maximum deceleration encounter by the jumper per each cord. Finally they have to answer the question which cord is the safest?



Exemplary data for two different elastic cords.

It would be very instructive for students to compare maximum force with weight of a "jumper" and then evaluate "overweight".

Hints and tips:

The stand should be well clamped to make a stable support.

The Force sensor should be calibrated in such a way that it displays 0 when the cord hangs on it.

The sampling rate for measuring force should be relatively high (200 samples per second).

Coach 6 Activity:

3.Safe bungee cord

ACTIVITY 4. BUNGEE CHALLENGE

Learning Objectives:

1. To be able to modify the model of accelerated motion to the model of the bungee jumping
2. To interpret motion graphs of $y(t)$, $v_y(t)$, $a_y(t)$.

APPLIED ICT TECHNOLOGY:
MODELLING

STUDENT LEVEL:
AGE 17

RECOMMENDED SETTINGS:
STUDENT ACTIVITY

Operational Skills:

- Creating a dynamical model
- Creating graphs from model variables
- Using the Simulation option

Procedural Skills:

- Active observation during executing model
- Analysing data using graphs
- Reading values/slopes
- Evaluating model quality

Activity method:

During the activity students have to create a model of the bungee jumping. The model of the constant accelerated motion with the acceleration of 10 m/s^2 is already placed in the Model window. Students have to finish the model by modifying the acceleration definition and adding forces acting on the jumper.

Using the final model they have to solve the problem of finding the appropriate bungee cord (actually its stiffness) to make a safe bungee jump.

Questions:

During creating the model the following questions could appear:

- How do you calculate the acceleration during the bungee motion?
- What are the forces acting during the jump?
- What is the net force acting on the jumper when the cord is not stretched yet ($y < L$)?
- What is the net force acting on the jumper when the cord is stretched ($y > L$)?

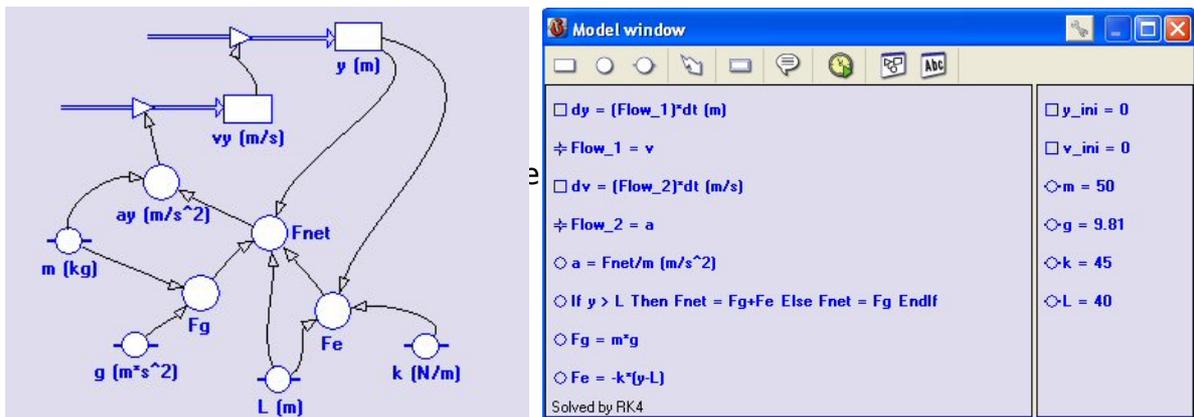
Analysing activities:

The model of the constant accelerated motion should be executed. After the execution the $y(t)$ graph should be interpreted. For the given stiffness k the maximum value of y should be read from the graph. Different graphs for different values of k should be created to determine the minimum value of k that will permit to survive the jump. This can be done by using the Simulation option.

Also the maximum "safe" value of k should be estimated. The topic Risks in Bungee jumping included in the chapter 2. Background theory gives some assumptions concerning how much acceleration the human body can withstand. Students can also use internet resources to find this out.

At the end of the activity students can improve their models by adding the air resistance force and observe and discuss the consequences of it.

Additionally other motions graphs $v(t)$, $a(t)$ and $a(y)$ could be interpreted.



Exemplary data: Coach 6 Graphical model of the bungee jumping and Coach 6 Equations model.

Coach 6 Activity:

4. Bungee challenge

FURTHER WORK - IDEAS FOR RELATED AND EXTENDED ACTIVITIES

1. Analysing data collected from a video of a bungee jump model e.g. a toy or a plastic bottle filled with water, recorded by students.
2. Analysing data of the motion of the bungee jumping model collected from the Motion detector or Accelerometer.
3. Energy loss while stretching a cord.